Circle Geometry

The BEST thing about Circle Geometry is....

you are given the Question and (mostly) the Answer as well.

You just need to fill in the gaps in between!
There are a number of definitions of the parts of a circle which you must know.

1. A circle consists of points which are equidistant from a fixed point (centre).
   The circle is often referred to as the circumference.

2. A radius is an interval which joins the centre to a point on the circumference.

3. A chord is an interval which joins two points on the circumference.

4. A diameter is a chord which passes through the centre.

5. An arc is a part of the circumference of a circle.

6. A segment is an area which is bounded by an arc and a chord. (We talk of the ALTERNATE segment, as being a segment on the OTHER side of a chord. We also refer to a MAJOR and a MINOR segment, according to the size.)

7. A semicircle is an area bounded by an arc and a diameter.

8. A sector is an area bounded by an arc and two radii.

9. A secant is an interval which intersects the circumference of a circle twice.

10. A tangent is a secant which intersects the circumference twice at the same point. (Usually we just say that a tangent TOUCHES the circle)

11. Concentric circles are circles which have the same centre.

12. Equal circles are circles which have the same radius.

13. Conyclic points are points which lie on the circumference of a circle.

14. A cyclic quadrilateral is a quadrilateral of which the vertices lie on the circumference of a circle.
The 21 theorems, which you need to be able to use, fit into a number of different categories. Here is a summary of them, for your reference.

1. a. Equal arcs subtend equal angles at the centre of the circle.
   b. If two arcs subtend equal angles at the centre of a circle, then the arcs are equal. (This is called a COROLLARY, or opposite, of the first theorem. Corollaries contain IF . . . . THEN statements)
   c. Equal chords subtend equal angles at the centre of a circle.
   d. Equal angles subtended at the centre of a circle cut off equal chords.

2. a. The angle at the centre of a circle is twice the angle at the circumference of a circle, standing on the same arc.
   b. Angles in the same segment of a circle are equal, standing on the same arc.
   c. The angle in a semicircle is a right angle. (This is a special case of a. above)

3. a. A perpendicular, from the centre of a circle to a chord, bisects the chord.
   b. An interval, from the centre of a circle to the midpoint of a chord, is perpendicular to the chord.
   c. Equal chords are equidistant from the centre of a circle.
   d. Chords, which are equidistant from the centre of a circle, are equal.

4. The products, of the intercepts of intersecting chords of a circle, are equal.
5. a. The opposite angles of a cyclic quadrilateral are supplementary.

b. If the opposite angles of a quadrilateral are supplementary, the quadrilateral is cyclic.

c. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

6. a. The tangent to a circle is perpendicular to the radius at the point of contact.

b. If the angle, between a radius and an interval at the point of contact on the circle, is a right angle, then the interval is a tangent.

c. Tangents to a circle, from an external point, are equal.

d. When two circles touch, the interval joining the centres of the circle passes through the point of contact. (Note: the circles could be embedded)

e. The angle between a tangent and a chord, at the point of contact, is equal to the angle in the alternate segment.

f. The square of the tangent from an external point is equal to the product of the intercepts of the secant from the same external point.

You need to be able to prove these theorems, as well as use them.

This topic also relies heavily on your ability to produce neat, accurate, concise and logical proofs for congruent triangle exercises. A good working knowledge of the properties of isosceles triangles is essential.
Illogical and sloppy proofs result in your losing marks in assessments and examinations.

If the Q is just a "find the value of" type, show enough working to convince the examiner that you actually worked it out.

Look for connections to Circle Geometry in other question.

For example: A and B are 2 points on the number plane. Find the locus of a point P which moves so that PA ⊥ PB.

Answer?

Another example: A circle with radius 5cm is drawn. From a point P, 8cm from the circumference of the circle, tangents are drawn to meet the circle at K and L. Find the length of the tangent PK, and find the length of the arc KL.

Answer? 11.76cm
Data: O is the centre of the circle

- $AB = CD$

- $ON \perp AB$ and $OM \perp CD$

Aim: (a) To prove $NX = MX$

(b) To prove $AX = DX$

Proof:

- [Proof steps here]
Data: O is the centre of the circle

Aim: To prove that

(a) \( p^\circ + q^\circ = 90^\circ \)
(b) \( r^\circ - p^\circ = 2q^\circ \)

Proof:
**Topic: Circle Geometry**

Data: ABC is an acute angled triangle inscribed within a circle with diameter AD.

BE \perp AC

DF \parallel BC

Aim: Prove that

(a) EGHC is cyclic

(b) \angle BAD = \angle FAC

(c) If BE is extended to meet the circumference at J, prove arc CJ = arc BD.

Proof:

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Here are some deductive geometry theorems which, while not strictly in the Ext 1 syllabus, are very useful to know. They have to do with the various centres of a triangle, but first you need to be familiar with a few terms.

1. The CENTROID (S) of a triangle is the point of intersection of the MEDIANS.
   It is also the centre of gravity of the triangle. The centroid is a third of the way along each median.

2. The CIRCUMCENTRE (O) of a triangle is the point of intersection of the perpendicular bisectors of the sides.

3. The INCENTRE (I) of a triangle is the point of intersection of the angle bisectors.

4. The ORTHOCENTRE (H) of a triangle is the point of intersection of the altitudes of a triangle.

5. The Centroid, the Orthocentre and the Circumcentre of a triangle are collinear. This line is called the EULER LINE.

6. The NINE-POINT CIRCLE. A circle can be drawn through the following points: (1) the midpoint of the sides of a triangle, (2) the points of segments intersection between the altitudes and the sides, and (3) the midpoints of the line between the vertices of the triangle and the intersection of the altitudes.
   (There are 3 points in each category) It is drawn below.

7. The centre of mass (X) of a SOLID is the point of intersections of the intervals joining a vertex to the centroid of the opposite side. The centre of mass is a quarter of the way along each of these intervals.
Another useful thing to remember is this: (it is a variation of the Intercept Theorem) If two triangles have the same length of base, and the same altitude, then they have the same area.

Here are some questions which use some of these theorems. Have a go and see if you can work them out. A diagram always makes it a lot easier.

1. P is the midpoint of the side AB of a parallelogram ABCD. BD cuts CP at Q, and CA at K. Prove that BQ = 2QK.

2. The sides BC, CA and AB of Δ ABC are produced by their own lengths to X, Y and Z respectively. Prove that the area Δ XYZ = 7 Δ ABC.

3. ABCD is a parallelogram. P is any point NOT inside the parallelogram. Q is the midpoint of PB. AQ is produced to R such that AQ = QR. Prove that RD and PC bisect each other.

4. ABC is an isosceles triangle with AB=AC. P is a point on the bisector of angle A. BP produced meets AC at Q, and CP produced meets AB at R. Prove that BCQR form a cyclic quadrilateral.
Proof for Question 4.

**Data:**

- $AB = AC$
- $AD$ bisects angle $A$. $P$ is on $PD$.
- $BP$ produced meets $AC$ at $Q$.
- $CP$ produced meets $AB$ at $R$.

**Aim:** To show that $BCQR$ is a cyclic quadrilateral.
Topic: Fun Stuff

1. Ceva’s theorem

On the (large) triangle ABC, choose a point X, somewhere nearish the middle of the triangle.

Construct AX, and extend it to meet BC at P.

Construct BX, and extend it to meet AC at Q.

Construct CX, and extend it to meet BC at R.

Measure AR, RB, BP, PC, CQ and QA.

Calculate $\frac{AR}{RB} \times \frac{BP}{PC} \times \frac{CQ}{QA}$

And the answer is ?????

Depending on the accuracy of your drawing and measuring, the answer is 1.

2. Ptolemy’s Theorem

ABCD are concyclic points, and AC & BD are the diagonals of the cyclic quadrilateral.

Measure AB, BC, CD and DA. Measure AC and BD.

Ptolemy’s Theorem states that the sum of the product of the opposite sides of a cyclic quadrilateral is equal to the product of the diagonals.

In other words, $AB \times CD + AD \times BC = AC \times BD$.

How nifty is that!!!!!