

Permutations & Combinations

Extension 1 Mathematics

HSC Revision

UOW

PERMUTATIONS AND COMBINATIONS: REVIEW

1. A "combination" lock has 4 dials each with 10 digits. How many possible arrangements are there?
2. How many numbers can be made using the digits 1 to 4, with no digit allowed to be repeated?
3. A girl wants to invite 9 friends to her tea party, but has room for only 5. In how many ways can she make the choice?
4. a) What is the probability of obtaining 4 Aces in a game of poker? (5 cards dealt from a pack of 52).
b) What is the probability of obtaining a flush? (all cards of the same suit)
5. A queue has 4 girls and 4 boys standing in a line. How many different arrangements are possible if:
 - a) the boys and girls stand anywhere?
 - b) boys and girls alternate?
 - c) 2 particular girls wish to stand together?
 - d) all the boys stand together?
 - e) a particular girl Mindy does not wish to stand next to Carrie?
6. In a train there are 8 seats, with 4 facing the front and 4 facing backwards.
If 5 people sit down in the carriage, how many different ways can they be seated?
 - a) If 2 of the people don't like sitting backwards, in how many ways can they be arranged?
 - b) Find the probability that two particular people will sit opposite each other.
7. From a group of 9 people, how many groups of 5 may be selected so as to include the youngest or the eldest but not both?
8. At a dinner table 6 people, 3 men and 3 women, are to sit around the table.
 - a) How many different arrangements are possible?
 - b) Find the probability that two particular friends sit together.
 - c) How many ways can 3 particular people sit together?
 - d) What is the probability that all the boys are seated together?
 - e) What is the probability that the boys and girls alternate?
 - f) In how many ways can two people sit opposite each other?
9. Find the number of different arrangements of the word MATHEMATICS that are possible.
Of these words what is the probability that if a word is selected at random
 - a) it begins with M and ends with S?
 - b) the two M's stay together?
 - c) the letters 'THE' stay together?
10. A city council consists of 6 Liberal and 5 Labor aldermen, from whom a committee of 5 is chosen. What is the probability that the Liberals have a majority?
11. a) How many 5-digit numbers can be made using the numbers 1 to 9 if no repetition is allowed?
b) How many 5-digit numbers greater than 70 000 can be formed?
c) What is the probability that a number selected at random contains 1, 2, 3, 4, 5
 - i) in ascending order
 - ii) in any order

12. a) How many different rearrangements of the letters REARRANGE are possible?
b) If one of these is chosen at random, what is the probability that the R's are together.
c) If a rearrangement is chosen at random from those with the 3 R's together, what is the chance that it begins and ends with E?
13. Three marbles are selected from a bag containing 4 white, 5 blue and 8 red marbles.
What is the probability that
a) two are white and one is blue?
b) they are all of the same colour?
c) there is one of each colour?
14. 12 keys are to be placed on a key ring. What is the number of ways of arranging the keys if
a) the keys are all different?
b) there are 3 identical locker keys and 4 identical car keys?
15. a) In how many ways can 3 horses finish 1st, 2nd and 3rd place in a race that consists of 8 horses?
b) If I choose 6 different selections of 3 horses at random, find the probability that I win the trifecta.
16. In how many ways can the 5 letters of the word CONIC be arranged in a line so that the 2 C's are separated by at least one other letter.
17. Five different Maths books, 4 different Science books and 3 different English books are arranged randomly on a shelf.
a) In how many ways can they be arranged?
b) What is the probability that books of the same subject are together?
18. If 4 men are to be selected from 6 men and 3 women are to be selected from 5 women, how many ways can these people be arranged in a line if
a) there is no restriction?
b) the four men remain together?
c) a man is at the beginning of the line?
d) a man is at the beginning of the line and a woman is at the end of the line?
e) the men and women alternate?
19. From a group of 6 boys and 4 girls, in how many ways can 2 boys and 2 girls be selected and arranged in a line?
20. If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can these books be arranged on a shelf if:
a) there are no restrictions?
b) the 4 Maths books remain together?
c) a Maths book is at the beginning of the shelf?
d) Maths and English books alternate?
e) a Maths book is at the beginning and an English book is in the middle of the shelf?

ANSWERS

1. 10^4 2. 64 3. 126 4. a) $1/54145$ b) 0.00198
5. a) 8! b) 1152 c) 10080 d) 2880 e) 30240
6. Total = 6720 a) 1440 b) $1/7$ 7. 70
8. a) 120 b) $48/120 = 2/5$ c) 36 d) $36/120 = 3/10$ e) $1/10$ f) 24
9. Total = 0.018 a) 0.18 b) 0.109
10. $281/462 = 0.608$ 11. a) 15120 b) 5040 c) (i) $1/15120$ (ii) $1/126$
12. a) 15120 b) $1/1260$ c) $1/21$ 13. a) $3/68$ b) $7/68$ c) $4/17$
14. a) 19958400 b) 138600 15. a) 336 b) $1/56$
16. 36 17. a) 12! b) 0.000216
18. . a) 756000 b) 86400 c) 432000 d) 216000 e) 21600
19. 2160
20. a) 756000 b) 86400 c) 432000 d) 21600 e) 216000

SOLUTIONS

1. 10^4 2. ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ 3. 9C_5 4. a) $({}^4C_4 \times {}^{48}C_1) / {}^{52}C_5$ b) $({}^{13}C_5 \times 4) / {}^{52}C_5$

5. a) $8!$ b) $4! 4! 2!$ c) $7! 2!$ d) $5! 4!$ e) $8! - 7! 2!$

6. Total = 8P_5 a) ${}^4P_2 \times {}^6P_3$ b) $({}^8P_1 \times 1 \times {}^6P_3) / {}^8P_5$ 7. ${}^7C_4 + {}^7C_4$

8. a) $(6 - 1)!$ b) $2! \times (5 - 1)! / 5!$ c) $3! \times (4 - 1)!$ d) $3! \times (4 - 1)! / 5!$ e) $3! 2! / 5!$ f) $4!$

9. Total = $11! / (2! 2! 2!)$ a) $(9! / (2! 2!)) / \text{Total}$ b) $(10! / (2! 2!)) / \text{total}$ c) $(3! 9! / (2! 2!)) / \text{total}$

10. $[{}^6C_3 \times {}^5C_2 + {}^6C_4 \times {}^5C_1 + {}^6C_5] / {}^{11}C_5$

11. a) 9P_5 b) ${}^3P_1 \times {}^8P_4$ c) (i) $1 / {}^9P_5$ (ii) ${}^5P_5 / {}^9P_5$

12. a) $9! / (3! 2! 2!)$ b) $(7! / (2! 2!)) / \text{total}$ c) $(5! / 2!) / (7! / (2! 2!))$

13. a) ${}^4C_2 \times {}^5C_1 / {}^{17}C_3$ b) $[{}^4C_3 + {}^5C_3 + {}^8C_3] / {}^{17}C_3$ c) ${}^4C_1 \times {}^5C_1 \times {}^8C_1 / {}^{17}C_3$

14. a) $(12 - 1)! / 2$ b) $(12 - 1)! / (2 \times 3! \times 4!)$

15. a) 8P_3 b) $6 / {}^8P_3$ 16. $3! \times 6$ 17. a) $12!$ b) $(5! 4! 3! 3!) / 12!$

18. a) ${}^6C_4 \times {}^5C_3 \times 7!$ b) ${}^6C_4 \times {}^5C_3 \times 4! \times 4!$ c) ${}^6C_4 \times {}^5C_3 \times {}^4P_1 \times 6!$

d) ${}^6C_4 \times {}^5C_3 \times {}^4P_1 \times {}^3P_1 \times 5!$ e) ${}^6C_4 \times {}^5C_3 \times 4! \times 3!$ or ${}^6P_4 \times {}^5P_3$

19. ${}^6C_2 \times {}^4C_2 \times 4!$

20. a) ${}^6C_4 \times {}^5C_3 \times 7!$ b) ${}^6P_4 \times {}^5C_3 \times 4!$ c) $6 \times {}^5C_3 \times {}^5C_3 \times 6!$ d) ${}^6P_4 \times {}^5P_3$

e) $6 \times 5 \times {}^5C_3 \times {}^4C_2 \times 5!$